Complex Networks: Structure and Functionality. I. Spectra; II. Equivalence; III. Exploration

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Lecture I considers inhomogeneous *Erdős-Rényi random graphs* on n vertices. We study the *empirical spectral distribution* of the *adjacency matrix* A_n in the limit as $n \to \infty$ in a regime that interpolates between sparse and dense. In particular, we show that the empirical spectral distribution of A_n when properly scaled converges to a deterministic limit weakly in probability. For the special case where the connectivity probability between two vertices has the *product property*, we give an explicit characterisation of the limit distribution. The result is applied to statistical inference of *sociability patterns in social networks*. (Based on joint work with Arijit Chakrabarty, Rajat Hazra and Matteo Sfragara.)

Lecture II considers random graphs subject to topological constraints. We compare two probability distributions on the set of simple graphs on n vertices induced by a given constraint: (1) The *microcanonical ensemble*, where the constraint is *hard*, i.e., has to be satisfied for every realisation of the graph; (2) The *canonical ensemble*, where the constraint is *soft*, i.e., has to be satisfied on average. We say that *breaking of ensemble equivalence* occurs in the limit as $n \to \infty$ when the *relative entropy* of the two ensembles per vertex (in the sparse regime), respectively, per edge (in the dense regime) is strictly positive. We present two examples of constraints where breaking of ensemble equivalence occurs, namely, when the constraint is on the degree sequence and when the constraint is on the total number of edges and triangles. The result is applied to model selection for real-world networks. (Based on joint work with Diego Garlaschelli, Michel Mandjes, Andrea Roccaverde, Tiziano Squartini and Nicos Starreveld.)

Lecture III considers the mixing time of random walks on random graphs. Many real-world networks, such as WWW, are dynamic in nature. It is therefore natural to study random walks on *dynamic random graphs*. We consider random walk on a random graph with prescribed degrees. We investigate what happens when at each unit of time a fraction α_n of the edges is randomly rewired, where n is the number of vertices. We identify *three regimes* for the mixing time in the limit as $n \to \infty$, depending on the choice of α_n. These regimes exhibit surprising behaviour. The results are relevant for Google PageRank. (Based on joint work with Luca Avena, Hakan Guldas and Remco van der Hofstad.)